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DEVELOPMENT OF A SIMPLIFIED DESIGN FORMULA FOR THE LOW FREQUENCY CUT-OFF OF A SMALL TWO VOLUME SILENCER

by

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ABSTRACT

One of the most difficult tasks in compressor suction or discharge muffler design is to successfully attenuate very low frequencies (typically the first and second harmonic of the operating speed; 60 and 120 Hz for a 3600 RPM compressor). Design formulas for the low frequency cut-off of two-volume silencer effectiveness are developed and some design implications are discussed. The work is based on the Helmholtz approximations.

NOMENCLATURE

V_1, V_2 = volumes [m^3]
p_2, p_2 = acoustic pressures relative to ambient [N/m^2]
T_1, T_2 = temperature [$^0 K$]
L_1, L_2 = corrected effective neck length [m]
$L_1 = L_{G1} + \sqrt{\pi A_1/2}$
$L_2 = L_{G2} + \sqrt{\pi A_2/2}$
L_{G1}, L_{G2} = geometric effective neck length [m]
A_1, A_2 = average cross-sectional areas of necks [m^2]
ρ_1, ρ_2 = average mass densities [Ns^2/m^4]
$\Delta V_1, \Delta V_2$ = dynamic change in volumes [m^3]
K_1, K_2 = bulk moduli [N/m^2]
D_1, D_2 = viscous damping coefficients [Ns/m^3]
C = average speed of sound [m/s]

INTRODUCTION

A frequently used configuration of a low pass filter muffler for small compressors and engines is a two volume muffler having a transfer pipe and tail pipe. While this or similar configurations have been analyzed frequently in the past, using various approaches such as lumped time domain models, four pole models based on the wave equation solution, and others, the approach to designing such a silencer has usually been to analyze a design configuration after the fact. The design method, is therefore, typically an iteration. One guesses the best proportions, analyzes the acoustic performance of the silencer, compares it to the desired performance, arrives at a new, hopefully improved design, analyzes again and so on. It is easily possible, if the designer is inexperienced, that the approach converges to an adequate design which is far from an optimal design. A second design method is automatic optimization. The problem with this is that it is often difficult to define which constraints have to be introduced. If not properly done by experienced designers, the muffler design may converge to strange proportions which may violate the physics of the theory which was the basis of the optimization program in the first place.

The third approach is to design the silencer in its approximate proportions using "rules of thumb" and experimentation. It has the potential short coming that it may become time consuming if an adequate initial design cannot be found quickly.

It is not the intent of this paper to make a case in favor of one of the three approaches. All three are of practical value. Rather, a simplified design formula is developed, following a philosophy of approach to small one volume engine mufflers developed by the authors, which will allow one to narrow the choice of several dimensional ratios by utilizing the concept of design by cut-off frequency, which is the frequency above which the silencer abates sound except for standing wave frequencies. This should be of value no matter which approach to design is used. Below the cut-off frequency, the silencer has two frequencies at which it may magnify sound. In order to accomplish this in a simple way, we confined ourselves to silencers of small enough dimensions so that at the cut-off frequency the associated wavelength is larger than approximately four times the largest muffler dimension (quarter wave criterion). This allowed us to employ the Helmholtz resonator simplifications.

The approach which will be presented allows one to start with the definition of the desired cut-off frequency of the silencer, select the dimensional ratios of a first workable design, and then to calculate the necessary tail pipe length. Subsequently this design can be improved by more sophisticated computer modeling procedures and/or experimentation.

GOVERNING EQUATIONS

This design is typically used in suction silencer design, but it can also be viewed as an approximation of a three volume muffler ending in an anechoic termination if the third volume just before the anechoic termination (for example, the shock loop which is often approximated as a pipe of infinite extension) is very large. Typically used two volume discharge muffler arrangements terminating in an "infinite" shock loop are not covered by the following, but similar design formulas can and have been developed by the authors.

Figure 1 is a schematic idealization. Neglecting the influence of mean velocity V_m, since V_m is typically much less than the speed of sound, it does not matter in what direction the mean flow is pointed. The valve (suction or discharge) provides a sum of harmonic volume velocity inputs, one of which is shown at the left.

The free body diagrams of the forces acting on the gas in the tubes are also shown in Figure 1. Any geometric shape that consists of a volume and a short neck and is filled with a compressible gas or fluid can be viewed as a Helmholtz resonator. Helmholtz realized that if a volume-neck combination is made to vibrate, it behaves essentially as if the neck is an incompressible plug that vibrates as a whole on the spring provided by the compressible gas in the volume. He found that he could analyze the resulting oscillation if he assumed that the compression process was linear. The relationship between the Helmholtz simplification and the wave equation was illustrated by Elson and Soedel [9].

Referring to the free body diagram for gas column 1 in Figure 1, the mass is

$$m_1 = L_1 A_1 \rho_1 \,, \tag{1}$$

the pressure p_1 in volume V_1 is (K_1 is the bulk modulus):

$$p_1 = -K_1 \frac{\Delta V_1}{V_1} \,, \tag{2}$$

and the change in volume V_1 is

$$\Delta V_1 = A_1 \xi_1 - A_o \xi_o \,. \tag{3}$$

However, ξ_o is usually not given, instead, we know that the input is a sinusoidally varying volume velocity

$$Q(t) = Q_n e^{j\omega_n t} = \{\text{either } Q_n \sin \omega_n t \text{ or } Q_n \cos \omega_n t\} \,. \tag{4}$$

Thus,

$$A_0 \xi_0 = \int_0^t Q_n e^{j\omega_n t} dt \,. \tag{5}$$

Substituting equation (5) into (3) gives

$$\Delta V_1 = A_1 \xi_1 - \int_0^t Q_n e^{j\omega_n t} dt . \tag{6}$$

Substituting into (2) gives

$$p_1 = \frac{-K_1 A_1 \xi_1}{V_1} + \frac{K_1}{V_1} \int_0^t Q_n e^{j\omega_n t} dt . \tag{7}$$

The pressure p_2 in volume V_2 is

$$p_2 = -K_2 \frac{\Delta V_2}{V_2} , \tag{8}$$

where

$$\Delta V_2 = A_2 \xi_2 - A_1 \xi_1 . \tag{9}$$

Thus

$$p_2 = \frac{-K_2 A_2 \xi_2}{V_2} + \frac{K_2 A_1 \xi_1}{V_2} . \tag{10}$$

Therefore, summing up all the forces in the free body diagram for gas column 1 gives

$$p_1 A_1 - p_2 A_2 - D_1 A_1 \dot{\xi}_1 - m_1 \ddot{\xi}_1 = 0 , \tag{11}$$

or

$$L_1 A_1 \rho_1 \ddot{\xi}_1 + D_1 A_1 \dot{\xi}_1 + \left[\frac{K_2}{V_2} + \frac{K_1}{V_1}\right] A_1^2 \xi_1 - \frac{K_2 A_2 A_1 \xi_2}{V_2} = \frac{K_2 A_1}{V_1} \int_0^t Q_n e^{j\omega_n t} dt . \tag{12}$$

Next, we consider the free body diagram of gas column 2 in Figure 1, and obtain

$$L_2 A_2 \rho_2 \ddot{\xi}_2 + D_2 A_2 \dot{\xi}_2 + \frac{K_2 A_2^2}{V_2} \xi_2 - \frac{K_2 A_1 A_2}{V_2} \xi_1 = 0 . \tag{13}$$

By defining volume velocities

$$Q_1 = A_1 \dot{\xi}_1 , \quad Q_2 = A_2 \dot{\xi}_2 , \tag{14,15}$$

and differentiating eqs. (12) and (13) with respect to time, we get

$$\ddot{Q}_1 + \frac{D_1}{L_1 \rho_1} \dot{Q}_1 + \left[\frac{K_2}{V_2} + \frac{K_1}{V_2}\right] \frac{A_1}{L_1 \rho_1} Q_1 - \frac{K_2 A_1}{V_2 L_1 \rho_1} Q_2 = \frac{K_1 A_1}{V_1 L_1 \rho_1} Q_n e^{j\omega_n t} , \tag{16}$$

$$\ddot{Q}_2 + \frac{D_2}{L_2 \rho_2} \dot{Q}_2 + \frac{K_2 A_2}{V_2 L_2 \rho_2} Q_2 - \frac{K_2 A_2}{V_2 L_2 \rho_2} Q_1 = 0 . \tag{17}$$

Defining

$$\omega_{11}^2 = \left[\frac{K_1}{V_1} + \frac{K_2}{V_2}\right] \frac{A_1}{L_1 \rho_1} \quad , \quad \omega_{22}^2 = \frac{K_2 A_2}{V_2 L_2 \rho_2} , \tag{18,19}$$

$$\omega_{12}^2 = \frac{K_2 A_1}{V_2 L_1 \rho_1} \quad , \quad \omega_{21}^2 = \frac{K_2 A_2}{V_2 L_2 \rho_2} \quad , \quad \omega_{01}^2 = \frac{K_1 A_1}{V_1 L_1 \rho_1} , \tag{20,21,22}$$

$$\zeta_1 = \frac{D_1}{2 L_1 \rho_1 \omega_{11}^2} \quad , \quad \zeta_2 = \frac{D_2}{2 L_2 \rho_2 \omega_{22}^2} , \tag{23,24}$$

we may finally write

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{Q}_1 \\ \ddot{Q}_2 \end{bmatrix} + \begin{bmatrix} 2\zeta_1 \omega_{11}^2 & 0 \\ 0 & 2\zeta_2 \omega_{22}^2 \end{bmatrix} \begin{bmatrix} \dot{Q}_1 \\ \dot{Q}_2 \end{bmatrix} + \begin{bmatrix} \omega_{11}^2 & -\omega_{12}^2 \\ -\omega_{21}^2 & \omega_{22}^2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = \begin{bmatrix} \omega_{01}^2 Q_n e^{j\omega_n t} \\ 0 \end{bmatrix} . \tag{25}$$

NATURAL FREQUENCIES

We set $\zeta_1 = \zeta_2 = 0$ and $Q_n = 0$. Eq. (25) becomes

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} \ddot{Q}_1 \\ \ddot{Q}_2 \end{bmatrix} + \begin{bmatrix} \omega_{11}^2 & -\omega_{12}^2 \\ -\omega_{21}^2 & \omega_{22}^2 \end{bmatrix} \begin{bmatrix} Q_1 \\ Q_2 \end{bmatrix} = 0 \tag{26}$$

At a natural frequency ω_n,

$$Q_1 = \overline{Q}_1 \sin \omega_n t \quad , \quad Q_2 = \overline{Q}_2 \sin \omega_n t \, , \tag{27,28}$$

where $\overline{Q}_1$ and $\overline{Q}_2$ are amplitudes of motion. Substituting eqs. (27) and (28) into eq (26) gives

$$\begin{bmatrix} -\omega_n^2 + \omega_{11}^2 & -\omega_{12}^2 \\ -\omega_{21}^2 & -\omega_n^2 + \omega_{22}^2 \end{bmatrix} \begin{bmatrix} \overline{Q}_1 \\ \overline{Q}_2 \end{bmatrix} = 0 \, . \tag{29}$$

This equation can only be zero if the determinant of the matrix is zero:

$$\begin{vmatrix} \omega_{11}^2 - \omega_n^2 & -\omega_{12}^2 \\ -\omega_{21}^2 & \omega_{22}^2 - \omega_n^2 \end{vmatrix} = 0 \, . \tag{30}$$

Solving this equation gives

$$\omega_{n1,2}^2 = \frac{(\omega_{11}^2 + \omega_{22}^2)}{2} \pm \sqrt{\left[\frac{(\omega_{11}^2 + \omega_{22}^2)}{2}\right]^2 - (\omega_{11}^2 \omega_{22}^2 - \omega_{12}^2 \omega_{21}^2)} \, . \tag{31}$$

RESPONSE RATIO

In steady state, the solutions of eq (25) must be of the form

$$Q_1 = \tilde{Q}_1 e^{j\omega t} \quad , \quad Q_2 = \tilde{Q}_2 e^{j\omega t} \, . \tag{32,33}$$

We obtain

$$\begin{bmatrix} (\omega_{11}^2 - \omega^2 + 2\zeta_1 \omega_{11} j\omega) & -\omega_{12}^2 \\ -\omega_{21}^2 & (\omega_{22}^2 - \omega^2 + 2\zeta_2 \omega_{22} j\omega) \end{bmatrix} \begin{bmatrix} \tilde{Q}_1 \\ \tilde{Q}_2 \end{bmatrix} = \begin{bmatrix} \omega_{01}^2 Q_n \\ 0 \end{bmatrix} \tag{34}$$

Solving for $\tilde{Q}_1$ and $\tilde{Q}_2$ and dividing the magnitude of $\tilde{Q}_2$ by Q_n gives the ratio of "output" to "input" volume velocity as

$$R = \frac{|\tilde{Q}_2|}{Q_n} = \frac{\left[\frac{\omega_{01}}{\omega_{11}}\right]^2}{\sqrt{\begin{gathered}\{\left[1 - \left[\frac{\omega}{\omega_{01}}\right]^2 \left[\frac{\omega_{01}}{\omega_{11}}\right]^2\right] \left[1 - \left[\frac{\omega}{\omega_{01}}\right]^2 \left[\frac{\omega_{01}}{\omega_{22}}\right]^2\right] \\ - \left[\frac{\omega_{01}}{\omega_{11}}\right]^2 \left[\frac{\omega_{11}}{\omega_{01}}\right]^2 - 4\zeta_1\zeta_2 \left[\frac{\omega}{\omega_{01}}\right]^2 \left[\frac{\omega_{01}}{\omega_{11}}\right] \left[\frac{\omega_{01}}{\omega_{22}}\right] \}^2 \\ + 4\{\zeta_2 \left[\frac{\omega}{\omega_{01}}\right] \left[\frac{\omega_{01}}{\omega_{22}}\right] \left[1 - \left[\frac{\omega}{\omega_{01}}\right]^2 \left[\frac{\omega_{01}}{\omega_{11}}\right]^2\right] + \zeta_1 \left[\frac{\omega}{\omega_{01}}\right] \left[\frac{\omega_{01}}{\omega_{11}}\right] \left[1 - \left[\frac{\omega}{\omega_{01}}\right]^2 \left[\frac{\omega_{01}}{\omega_{22}}\right]^2\right] \}^2\end{gathered}}} \tag{35}$$

and the phase angle ϕ_2 is

$$\phi_2 = \tan^{-1} \frac{2\zeta_2 \left[\frac{\omega}{\omega_{22}}\right] \left[1 - \left[\frac{\omega}{\omega_{11}}\right]^2\right] + 2\zeta_1 \left[\frac{\omega}{\omega_{11}}\right] \left[1 - \left[\frac{\omega}{\omega_{22}}\right]^2\right]}{\left[1 - \left[\frac{\omega}{\omega_{11}}\right]^2\right] \left[1 - \left[\frac{\omega}{\omega_{22}}\right]^2\right] - \left[\frac{\omega_{21}\omega_{12}}{\omega_{11}\omega_{22}}\right]^2 - 4\zeta_1\zeta_2 \left[\frac{\omega^2}{\omega_{11}\,\omega_{22}}\right]}, \quad (36)$$

where, for $K_1 = K_2 = \rho c^2$, $\rho_1 = \rho_2$, we have

$$\frac{\omega_{12}}{\omega_{01}} = \sqrt{\frac{V_1}{V_2}}, \; \frac{\omega_{01}}{\omega_{11}} = \sqrt{\frac{1}{1 + \frac{V_1}{V_2}}}, \; \frac{\omega_{01}}{\omega_{22}} = \sqrt{\left[\frac{A_1}{A_2}\right] / \left[\left[\frac{V_1}{V_2}\right]\left[\frac{L_1}{L_2}\right]\right]}, \quad (37,38,39)$$

$$\omega_{11} = C\sqrt{\left[\frac{1}{V_1} + \frac{1}{V_2}\right]\frac{A_1}{L_1}}, \; \omega_{22} = C\sqrt{\left[\frac{1}{V_2}\right]\frac{A_2}{L_2}} = \omega_{21}, \quad (40,41)$$

$$\omega_{12} = C\sqrt{\left[\frac{1}{V_2}\right]\frac{A_1}{L_1}}, \quad \omega_{01} = C\sqrt{\left[\frac{1}{V_1}\right]\frac{A_1}{L_1}}. \quad (42,43)$$

The muffler will attenuate sound whenever $R < 1$. There will be two lower frequency regions where the muffler will potentially amplify sound. These regions correspond approximately to the two resonance frequencies of the muffler.

CUT-OFF FREQUENCY

It is of interest to calculate the cut-off frequency above which the muffler will be generally effective in attenuating pulsations. For this purpose, we take the situation which gives the highest value of cut-off frequency for a given set of parameters, namely when $\zeta_1 = \zeta_2 = \zeta = 0$. Setting $R = 1$ gives, from eq. (35).

$$1 = \frac{\pm\left[\frac{\omega_{01}}{\omega_{11}}\right]^2}{\left[1 - \left[\frac{\omega}{\omega_{01}}\right]^2 \left[\frac{\omega_{01}}{\omega_{11}}\right]^2\right] \left[1 - \left[\frac{\omega}{\omega_{01}}\right]^2 \left[\frac{\omega_{01}}{\omega_{22}}\right]^2\right] - \left[\frac{\omega_{12}}{\omega_{11}}\right]^2}, \quad (44)$$

where the $\pm$ sign comes from the fact that the denominator is $\pm\sqrt{(\cdots)^2} = \pm(\cdots)$. Expanding gives

$$\left[\frac{\omega}{\omega_{01}}\right]^4 - B\left[\frac{\omega}{\omega_{01}}\right]^2 + C_{1,2} = 0, \quad (45)$$

where

$$B = \left[\frac{\omega_{11}}{\omega_{01}}\right]^2 + \left[\frac{\omega_{22}}{\omega_{01}}\right]^2 = 1 + \frac{V_1}{V_2} + \left[\frac{A_2}{A_1}\right]\left[\frac{V_1}{V_2}\right]\left[\frac{L_1}{L_2}\right], \quad (46)$$

$$C_{1,2} = \left[\frac{\omega_{11}}{\omega_{01}}\right]^2 \left[\frac{\omega_{22}}{\omega_{01}}\right]^2 - \left[\frac{\omega_{12}}{\omega_{01}}\right]^2 \left[\frac{\omega_{22}}{\omega_{01}}\right]^2 \pm \left[\frac{\omega_{22}}{\omega_{01}}\right]^2 = 0,2\left[\frac{V_1}{V_2}\right]\left[\frac{L_1}{L_2}\right]\left[\frac{A_2}{A_1}\right], \quad (47)$$

and where $C_1 = 0$ is the value for the minus sign and C_2 is the value for the plus sign.

The solution is

$$\left[\frac{\omega}{\omega_{01}}\right]^2 = \frac{B \pm \sqrt{B^2 - 4C_{1,2}}}{2}. \tag{48}$$

For example, for $A_1/A_2 = 1$, $L_1/L_2 = 1$, $V_1/V_2 = 1$, we obtain $B = 3$, $C_1 = 0$ and $C_2 = 2$. There are, therefore, four values of ω/ω_{01} at which $R = 1$: $(\omega/\omega_{01}) = 0$ and 1.73, obtained from $C_1 = 0$, and $(\omega/\omega_{01}) = 1.0$ and 1.414, obtained from $C_2 = 2$. Or, $R = 1$ exists when $\omega = 0$, ω_{01}, $1.41\omega_{01}$ and $1.73\ \omega_{01}$.

Eq. (48) is perhaps more useful if eqs. (46) and (47) for $C_1 = 0$ are substituted ($C_1 = 0$ determines the highest cut-off frequency, $\omega = 1.73\,\omega_{01}$). This gives the condition that all frequency bands are attenuated whose frequency is higher than the cut-off frequency ω_c for the $C_1 = 0$ case:

$$\omega > \omega_C, \tag{49}$$

where

$$\omega_C = \omega_{01}\sqrt{B} = \omega_{01}\left[1 + \frac{V_1}{V_2} + \left[\frac{A_2}{A_1}\right]\left[\frac{V_1}{V_2}\right]\left[\frac{L_1}{L_2}\right]\right]^{1/2}. \tag{50}$$

Changing ω_C[rad/s] to f_C [Hz] and solving for L_1 gives

$$L_1 = \frac{A_1}{V_1}\left[\frac{C}{2\pi f_c}\right]^2\left[1 + \left[\frac{V_1}{V_2}\right] + \left[\frac{A_2}{A_1}\right]\left[\frac{V_1}{V_2}\right]\left[\frac{L_1}{L_2}\right]\right]. \tag{51}$$

For pipe length that are much larger than $(1/2)\sqrt{\pi A}$, we may set $L_1 = L_{G1}$ and $L_2 = L_{G2}$. Thus, if "best" ratios (V_1/V_2), (A_2/A_2), (L_1/L_2) can be rationally selected, and A_1, V_1 and f_C can be selected, L_1 can be calculated.

RESULTS AND DISCUSSION

In order to understand how muffler dimensions should be selected, a large variety of cases were selected and the logarithm of the ratio R was plotted versus the ratio of the volume flow oscillation input frequency to a nondimensionalizing frequency ω_{01}.

Figure 2 shows the influence of damping, which is for most mufflers an empirical parameter. Selecting $\zeta_1 = \zeta_2$, $A_1/A_2 = 1$, $L_1/L_2 = 1$ and $V_1/V_2 = 1$, and assuming an average speed of sound, results in three curves which differ from each other mainly by the amplitudes of the two resonance peaks. The muffler is only effective in regions ω/ω_{01} where the logarithm of the transmission ratio is negative. Ignoring the small stretch between the two peaks where this is the case, the main region of effectiveness is for values of ω/ω_{01} greater than the cut-off frequency, defined by this graph.

Next, damping was held constant, as were all other parameters except for the ratio of inertia block pipe length L_1 to tailpipe length L_2. It can be seen from Figure 3 that this ratio should be as small as practically feasible, but at the same time the absolute length L_1 should also be as large as possible because L_1 is part of the nondimensionalizing frequency ω_{01}. A practical recommendation is probably to make L_1 as large as feasible and then make L_2 twice as long.

Figure 4 shows what happens when the volume ratio V_1/V_2 is varied. Also, because it is required that the absolute value of V_1 be as large as possible since it is part of ω_{01}, the result is that the total muffler volume $V_1 + V_2$ should be as large as possible. At this point in time it is recommended to make the two volumes equal, $V_2 = V_1$. The volume V_1 should ideally be several times the displacement volume of the compressor. More work is necessary to define the minimum required volume V_1, probably in conjunction with modeling the compressor itself.

Figure 5 shows what happens when the ratio A_1/A_2 is varied. This plot is not of great consequency since flow resistance considerations usually demand that A_1 should be equal to at least the maximum valve flow area and that $A_2 \doteq A_1$.

Figure 6 shows the ratio R when "best" and "worst" ratios of volumes, tube length and crossectional areas are used in the range of the previous examples. It is needless to point out that "best" ratios are subject to design constraints. The most severe constraint is usually that the overall geometric size of mufflers is severly limited.

It is obvious from the equations and figures that as lower f_C can be selected, as better it is for the average noise attenuation. The fundamental and first few harmonics corresponding to compressor speed and number of cylinders are difficult to attenuate since f_c will have to be correspondingly low. Very low values of f_c will lead to large L_1 selections. The crossectional area A_1 is determined essentially by the maximum compressor valve flow area. It cannot be much smaller. The first volume V_1 is determined by the back pressure on the valves, which is generated by the muffler. There is a minimum value for each valve design, which can be determined only experimentally or by a more complete computer simulation program. As a first rule of thumb, V_1 should be about two times the displacement volume of the compressor, but may have to be smaller due to space limitation.

A possible design procedure sequence is as follows:

(a) Select volume V_1 of the first muffler chamber as large as possible (as initial guess about two times the compressor displacement volume). The shape of the volume is relatively unimportant.

(b) Select the crossection of the inertia block tube A_1, and also the crossection of the tail tube A_2, to be approximately equal to or larger than the maximum valve flow area of the compressor. The tubes do not have to be circular. They may also deviate from being cylindrical. In this case the crossections are average values.

(c) Select the cut-off frequency f_C of the muffler. It should be as low as possible. What it should be depends on the compressor noise or gas pulsation spectrum.

(d) Select, if possible, $V_2 \cong V_1$. The shape of V_2 is relatively unimportant.

(e) Select the length of the tail tube to be at least twice that of the inertia block tube, $L_2 \cong 2L_1$. Tubes do not have to be straight.

(f) Calculate the length of inertia block pipe L_1 from eq. (51). Use an average speed of sound C which corresponds to the average temperature in the muffler.

(g) Build a prototype muffler.

(h) Measure the achieved noise reduction.

(i) Measure if there is significant power loss.

(j) If low frequency pulsations are still too high, lower the cut-off frequency f_c.

(k) If there is too much power loss, increase the first volume V_1 (if possible). But readjust L_1 (recalculate) so that the cut-off frequency is not changed.

(l) Go back to (g) and iterate between theory and experiment.

Note that this is only a very approximate approach which will allow one to design into the direction of low frequency pulsation reduction. There may be higher frequency noise bands where the silencer is not very effective because standing waves are excited in the tailpipe and other parts of the muffler. Additional experimental work and theoretical approaches, not discussed here, will have to be applied in this case.

ACKNOWLEDGEMENT

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FIGURES

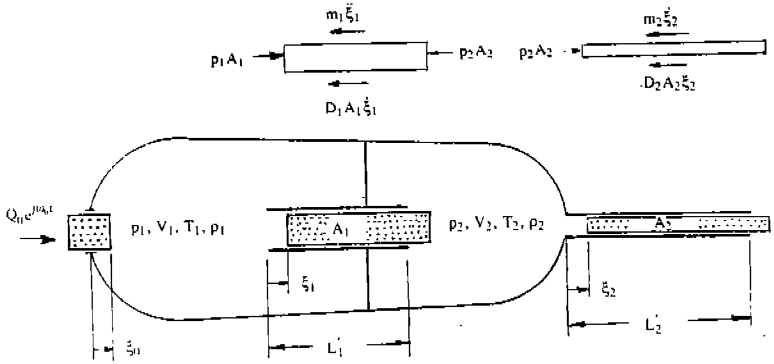

Figure 1. Two volume low pass filter muffler.

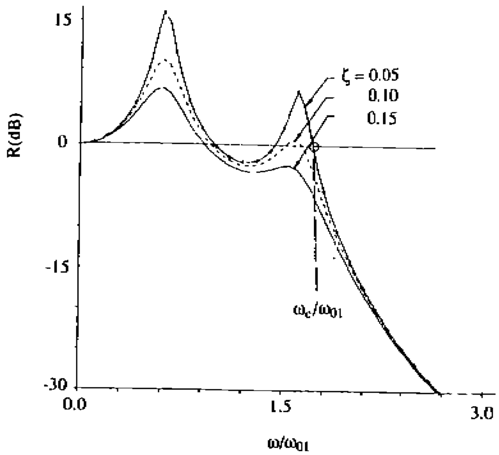

Figure 2. Influence of damping for $A_1/A_2 = 1$, $L_1/L_2 = 1$ and $V_2/V_2 = 1$.

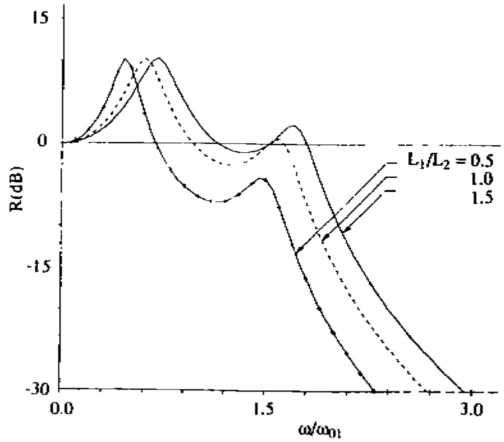

Figure 3. Influence of ratio of inertia block pipe length to tail pipe length for $\zeta = 0.1$, $A_1/A_2 = 1$ and $V_1/V_2 - 1$.

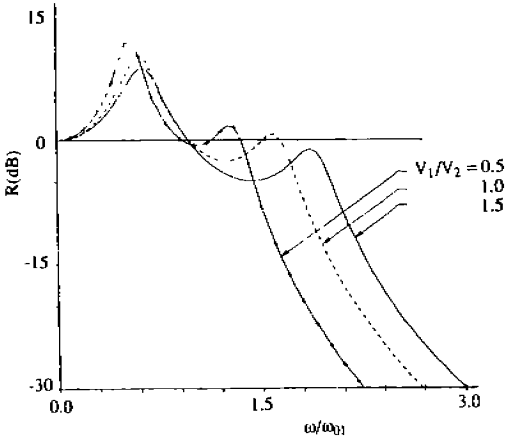

Figure 4. Influence of ratio of volumes for $\zeta = 0.1$, $A_1/A_2 = 1$ and $L_1/L_2 = 1$.

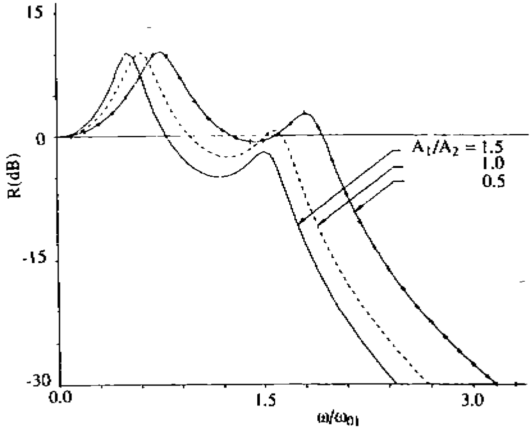

Figure 5. Influence of ratio of pipe cross-sectional area for $\zeta = 0.1$, $V_1/V_2 = 1$ and $L_1/L_2 = 1$.

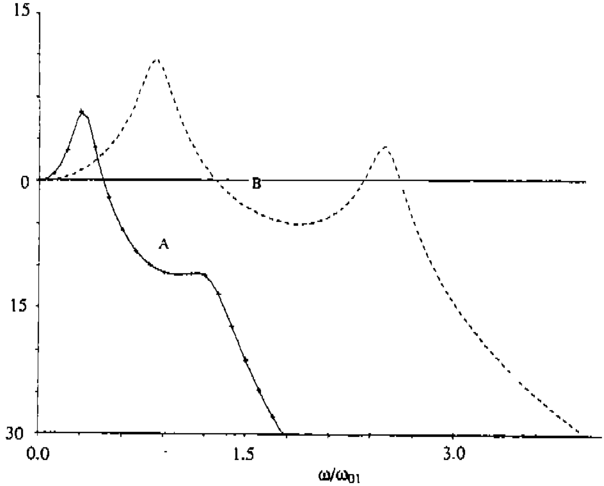

Figure 6. This figure illustrates how the two volume low pass filter muffler can vary in effectiveness when "best" ratios or "worst" ratios are selected simultaneously. Curve A is for $V_1/V_2 = 0.5$, $L_1/L_2 = 0.5$, $A_1/A_2 = 1.5$, $\zeta = 0.15$, curve B is for $V_1/V_2 = '.5$, $L_1/L_2 = 1.5$, $A_1/A_2 = 0.5$, $k\zeta = 0.05$. However, it has to be realized that the muffler of curve A has a much larger total volume ($(3V_1)$) than the muffler of curve B ($1.67V_1$).